Boğaziçi University Department of Physics

Phys 311/407

Summer 2013

Problem Set #7

Reading: Rohlf Chapter 6 Useful integrals:

$$\int_{-\infty}^{\infty} e^{-(ax^2 + bx)} = \sqrt{\frac{\pi}{a}} e^{b^2/4a} \quad a > 0$$
$$\int_{-\infty}^{\infty} x^2 \cdot e^{-(x/a)^2} = \frac{a^3}{2}\sqrt{\pi} \quad a > 0$$

Problem 1:

In class, we stated that $\Delta x \cdot \Delta p_x = \hbar/2$ for a Gaussian wave function by hand-waving. In this question you will show this quantitatively. Consider the wave function $\psi(x) = Ce^{-\frac{1}{2}(x/\sigma)^2}$ for a particle:

- a) Find C such that the probability distribution is properly normalized,
- b) Find g(k),
- c) Find the average position of the particle, $\langle x \rangle = \int dP \cdot x$, where P is the probability function obeying $dP/dx = |\psi(x)|^2$
- d) Find the average x^2 ; $\langle x^2 \rangle = \int dP \cdot x^2$,
- e) Find Δx ; $(\Delta x)^2 = \int P \cdot (x \langle x \rangle)^2$, (Hint: you do not to evaluate this integral; you can use the results found in previous parts.)
- f) Using the symmetry between $\psi(x)$ and g(k) functions, write down Δp_x without evaluating any integral,
- g) Show that $\Delta x \cdot \Delta p_x = \hbar/2$.

Problem 2: – Rohlf 5.22 – Modified

Even a professional musician with "perfect pitch" will have trouble identifying the pitch of a note if the duration is too short. Why? State your reasoning clearly, draw sketch of the waves if needed.

Problem 3:

If a particle can exist either at x = -L or at x = L with equal probability, what is Δx ?

Problem 4: – Rohlf 5.20 – Modified

Consider an electron and a proton each confined to a volume of 10^{-30} m³. Which particle has a larger minimum kinetic energy? Why? State your reasoning clearly.