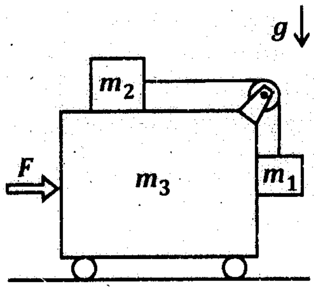
Time 120 min **Phys 101 Final Exam** T. Akdogan, T. Neyzi Fall'09

No:	Name:	Total Grade:	Grade:

***Question* 1 :** The cart in the figure is being pushed by a horizontal force as shown in the figure. Assume all surfaces, wheels, and pulley are frictionless, and the pulley is massless.

a) What horizontal force must be applied to the cart shown in figure so that the blocks remain stationary relative to the cart?

b) Draw the free body diagram of m_3 only.



***Answer* :**

FBD for $m_1+m_2+m_3$ all together.

N_T, F, $(m_1+m_2+m_3)g$ $\Rightarrow A$

$$\frac{F}{m_1+m_2+m_3} = A \,//$$

m_2: N_2, T, $m_2 g$ $\Rightarrow A$

$T = m_2 A$

$T = m_1 g$

m_1: T, N_1, $m_1 g$ $\Rightarrow A$

$m_2 A = m_1 g$

$$A = \frac{m_1 g}{m_2} = \frac{F}{m_1+m_2+m_3}$$

a) $\therefore$ $\boxed{F = (m_1+m_2+m_3)\dfrac{m_1 g}{m_2}}$ //

b) : m_3 only

N_T, F, T, N_1, $m_3 g$, N_2, T $\Rightarrow A$

T, F_p, T

pulley!

check:

$$F - \underset{m_2 A}{\cancel{T}} - \underset{m_1 A}{\cancel{N_1}} = m_3 A \quad \text{ok}$$

$$N_T = m_3 g + \underset{m_2 g}{\cancel{N_2}} + \underset{m_1 g}{\cancel{T}} \quad \text{ok} \,//$$

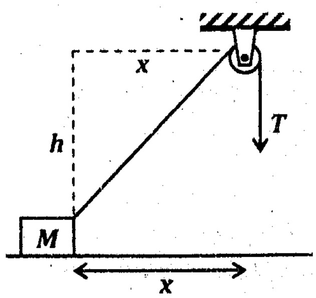
Grade :

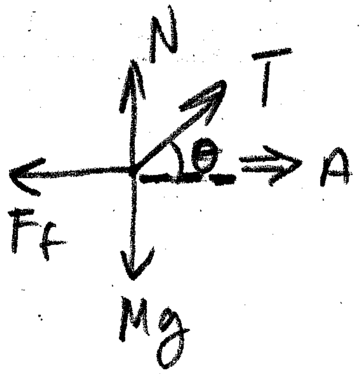
Question **2 :** A block of mass $\boldsymbol{M}$ is accelerated across a rough surface as shown in the figure. The tension $\boldsymbol{T}$ in the cord is maintained to be constant, and the pulley is at height $\boldsymbol{h}$ above the top of the block. The coefficient of kinetic friction is μ. The pulley and the cord are massless.

a) Draw the free body diagram for the block,

b) Find the acceleration of the block as a function of $\boldsymbol{x}$.

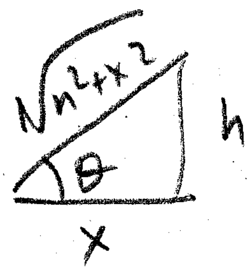
Answer **:**





$$T\cos\theta - F_f = MA$$
$$T\sin\theta - Mg + N = 0$$

$$T\cos\theta - \mu(Mg - T\sin\theta) = MA$$
$$\frac{T}{M}(\cos\theta + \mu\sin\theta) - \mu Mg = MA$$



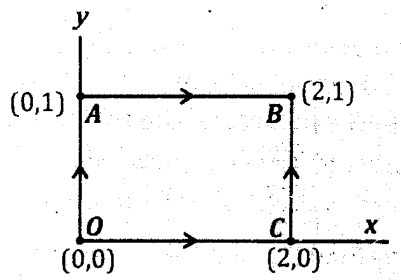
$$\frac{T}{M}\left(\frac{x}{\sqrt{h^2+x^2}} + \frac{\mu h}{\sqrt{h^2+x^2}}\right) - \mu g = A$$

$$\boxed{\frac{T}{M}\left(\frac{x+\mu h}{\sqrt{x^2+h^2}}\right) - \mu g = A}$$

Grade :

Question 3 : A force is given by $\vec{F}(x, y) = k_1 xy\,\hat{i} + k_2 x^2\,\hat{j}$ (k_1 and k_2 are positive constants). Find the work done by this force when a particle moves :

a) from ***O*** to ***A*** ,
b) from ***A*** to ***B*** ,
c) from ***O*** to ***C*** ,
d) from ***C*** to ***B*** , along the four sides of the rectangular path shown in the figure.
e) Find the relationship between k_1 and k_2 if $\vec{F}$ is a conservative force.



Answer :

$$\int \vec{F}\cdot d\vec{r} = W = \int (k_1 xy, k_2 x^2)\cdot(dx, dy)$$

a) $O \to A$ $\quad dx = 0 \quad x = 0 \quad W = 0$

b) $A \to B$ $\quad dy = 0 \quad y = 1 \quad \int (k_1 x, k_2 x^2)(dx, 0)$

$$\frac{k_1 x^2}{2}\Big|_0^2 = 2k_1 \,//$$

c) $O \to C$ $\quad dy = 0 \quad y = 0 \quad W = 0$

d) $C \to B$ $\quad dx = 0,\ x = 2 \quad \int (2k_1 y, k_2 4)(0, dy)$

$$= 4k_2 y\Big|_0^1$$

$$= 4k_2 \,//$$

$2k_1 = 4k_2$

e) $k_1 = 2k_2 \,//$ conservative

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check $\quad \vec{F} = (\underbrace{2k_2 xy}_{-\frac{dU}{dx}} ; \underbrace{k_2 x^2}_{-\frac{dU}{dy}}) \qquad U = -k_2 x^2 y \,//$

$\Delta U = -W \Big\} \quad (0,0) \to (2,1) \qquad U(0,0) = 0$

$U(2,1) = -k_2 4 \,//$

Grade :

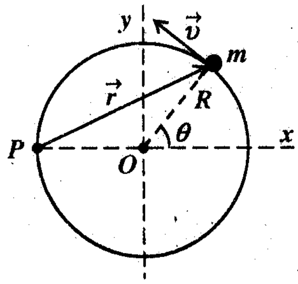
***Question* 4** : A particle of mass $\boldsymbol{m}$ moves in a circle of radius $\boldsymbol{R}$, such that $\boldsymbol{\theta = kt^2}$ (where $\boldsymbol{k}$ is a positive constant), as shown in the figure.

a) Find vector, $\vec{r}$ shown in the figure.

b) Find velocity vector, $\vec{v}$,

c) Find the linear momentum, $\vec{p}$,

d) Find the angular momentum $\vec{L}$ with respect to point $\boldsymbol{P}$.



***Answer* :**

$$\vec{r}_p = \vec{r}_0 + \vec{R}$$
$$= (R\cos\theta, R\sin\theta) + (R, 0)$$

a) $\vec{r} = (R + R\cos kt^2, R\sin kt^2)$ //

b) $\vec{v} = \frac{d\vec{r}}{dt} = (-2tkR\sin kt^2, 2ktR\cos kt^2)$

c) $\vec{p} = m\vec{v} = (-2tkRm\sin kt^2, 2ktRm\cos kt^2)$

d) $\vec{L}_p = \vec{r}_p \times m\vec{v}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ R+R\cos kt^2 & R\sin kt^2 & 0 \\ -2ktRm\sin kt^2 & 2ktRm\cos kt^2 & 0 \end{vmatrix}$$

$$= (0, 0, 2ktR^2m(1+\cos kt^2)\cos kt^2 + 2ktR^2m\sin^2 kt^2$$

$$= (0, 0, 2ktR^2m\cos kt^2 + 2ktR^2m)$$

$$= 2ktR^2m\,(0, 0, 1+\cos kt^2)$$ //

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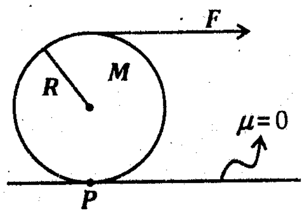
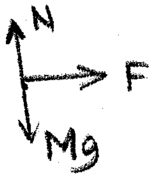
***Question* 5 :** A spool of wire of mass M and radius R is unwound under a constant horizontal force $\vec{F}$ as shown in the figure. Assuming the spool is a uniform solid cylinder, and the surface is frictionless,
a) Find the acceleration of the center of mass, $\vec{a}_{cm}$,
b) the angular acceleration, α, about the center of mass,
c) the linear acceleration of point P on the spool which is the contact point between the cylinder and the floor. [$I_{cylinder} = MR^2/2$.]



***Answer*:**

a) FBD

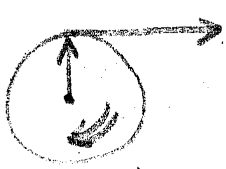


$\Rightarrow a_{cm}$

$F = Ma_{cm}$

$\boxed{\frac{F}{M} = a_{cm}}$ //

b)

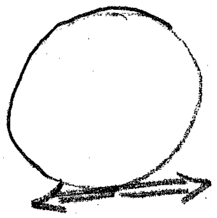


in +z //

$F\cancel{R} = \frac{MR^{\cancel{2}}}{2}\alpha$

$\boxed{\frac{2F}{MR} = \alpha}$ //

c)



$a_{cm} = F/M$

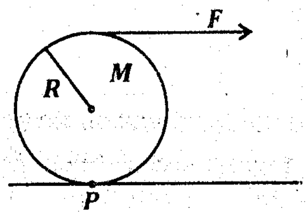
$\alpha R = \frac{2F}{m}$

$\Leftarrow F/m$

$\boxed{\text{Net } a_p = F/m \ (\Leftarrow) \text{ direction}}$

Grade :

Question 6 : A spool of wire of mass $\boldsymbol{M}$ and radius $\boldsymbol{R}$ is unwound under a constant horizontal force $\vec{F}$ as shown in the figure. Assuming the spool rolls without slipping:

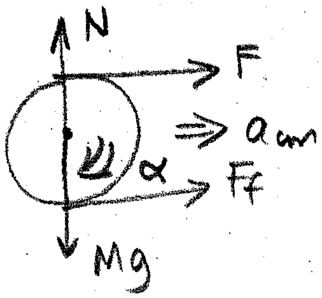


a) Find the acceleration of the center of mass, $\vec{a}_{cm}$,

b) the angular acceleration, $\boldsymbol{\alpha}$, about the center of mass,

c) the linear acceleration of point $\boldsymbol{P}$ on the spool which is the contact point between the cylinder and the floor. $[I_{cylinder} = MR^2/2\ .]$

Answer **:**



$$F + F_f = M a_{cm}$$

$$a_{cm} = R\alpha$$

$$F\cancel{R} - F_f \cancel{R} = \frac{M R^{\cancel{2}}}{2}\alpha$$

$+$

$$2F = \frac{3 M a_{cm}}{2}$$

a) $$\frac{4F}{3M} = a_{cm}$$ //

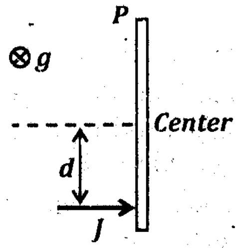
b) $$\frac{4F}{3MR} = \alpha$$ //

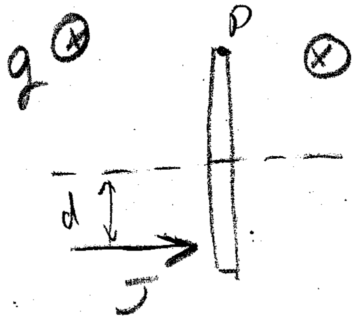
c) $$a_P = 0 \iff \frac{4F}{3M} = a_{cm}$$

$$R\alpha = \frac{4F}{3M}$$

Grade:

Question 7 : A stick of length ***L*** and mass ***m*** lies on a frictionless horizontal table on which it is free to move in any way. The stick is hit with impulse ***J*** apllied perpendicularly. Just after the impulse, find the following quantities:
a) The velocity of the center of mass of the stick.
b) The angular speed of the stick about its center of mass.
c) The net velocity of one of the end points ***P*** of the stick. (See figure.)
Answer :





$$\frac{d\vec{p}}{dt} = \vec{F}$$

$$\int d\vec{p} = \int \vec{F}\,dt \qquad \longrightarrow +x$$

x-comp: $p_f - p_i = J$

$$m v_{cm} = J$$

a) $v_{cm} = J/m$ //

b) out +z

$$\frac{d\vec{L}_{cm}}{dt} = \vec{\tau}_{cm}$$

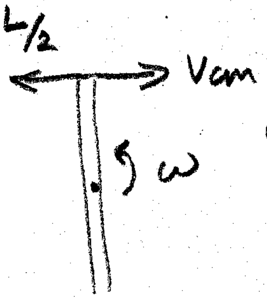
$$\Delta L = \int (F \cdot d)\,dt$$

z comp

$$\frac{mL^2}{12}\omega = J d$$

$$\omega = \frac{12 J d}{m L^2} \quad //$$

c)



$$\longleftarrow \qquad \longrightarrow$$

$$6\,\frac{\cancel{12}Jd}{mL^{\cancel{2}}}\,\frac{\cancel{L}}{\cancel{2}} \qquad J/m$$

$$\frac{6Jd}{mL} \qquad J/m$$

$$\longleftarrow \qquad \longrightarrow$$

$$\vec{v}_P = \frac{J}{m}\left(1 - \frac{6d}{L}\right)$$

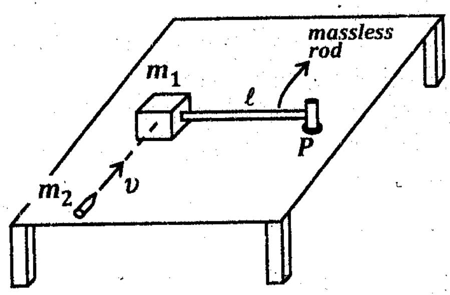
$$0 < d < \frac{L}{2}$$

Grade :

***Question* 8** : A wooden block of mass m_1 resting on a frictionless horizontal surface is attached to a rigid rod of length L and of a negligible mass (see figure). The rod is pivoted at the other end. A bullet of mass m_2 traveling parallel to the horizontal surface and perpendicular to the rod with speed v hits the block and becomes embedded in it.

a) Find the magnitude and direction of the angular momentum of the bullet-block system.

b) What fraction of the original kinetic energy is converted into internal energy in the collision.



Answer :

$m_1 + m_2$ // $\vec{L}$ is in inwards ⊥ to the plane of the surface.

$\vec{L}_i = m_2 \ell v \hat{k}$

$\vec{L}_f = (m_1 + m_2)\ell u$

$\vec{L}_i = \vec{L}_f$ //

$$m_2 \ell v = [(m_1 + m_2)\ell^2]\omega$$

$$\frac{m_2 v}{(m_1+m_2)\ell} = \omega \quad //$$

$$E_i = \frac{1}{2} m_2 v^2 \qquad E_i > E_f$$

$$E_f = \frac{1}{2}[(m_1+m_2)\ell^2]\omega^2$$

$$|\Delta E| = \text{converted energy} = \frac{1}{2} m_2 v^2 - \frac{1}{2}(m_1+m_2)\ell^2\omega^2$$

$$\frac{|\Delta E|}{E_i} = \frac{\cancel{\frac{1}{2}} m_2 v^2 - \cancel{\frac{1}{2}}(m_1+m_2)\ell^2\left(\frac{m_2^2 v^2}{(m_1+m_2)^2 \ell^2}\right)}{\cancel{\frac{1}{2}} m_2 v^2}$$

$$= \frac{m_2 v^2 - m_2^2 v^2 / m_1 + m_2}{m_2 v^2}$$

$$= 1 - m_2/m_1 + m_2 = m_1/(m_1 + m_2)$$